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Continuous "System-Level" Scale for Comparing Laser Gain Media

by Jeffrey O. White

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14. ABSTRACT
Several quantities are proposed for facilitating a quantitative comparison of laser gain media, operating temperatures, pump, and laser wavelengths. They are based on the occupation probability of absorbing and emitting pump and laser levels. The "system level," ℓ, has a numerical value coinciding with conventional usage of the terms two-, three-, and four-level system. The "occupation factor" f_0 has a value from –1 to +1, and is appropriate for describing an optical amplifier in the small signal regime. f_1 also ranges from –1 to +1, and is appropriate for describing an amplifier in the large signal regime, e.g., a laser. The physical significance is that for $\ell > 2$, $f_0 > 0$, or $f_1 > 0$, the laser beam gains photons at the expense of the pump beam, in steady state. For $\ell < 2$, $f_0 < 0$, or $f_1 < 0$, the opposite occurs. The proposed definition is general enough to apply to many types of gain media, but is particularly useful for comparing systems with discrete levels, pumped with a narrow-band source, in near-resonance with the laser wavelength. Several low-quantum-defect combinations of pump and laser wavelengths are analyzed for Er^{3+}, Nd^{3+}, and Yb^{3+} in YAG, as a function of temperature.

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Quasi-three-level lasers, four-level lasers

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1. Summary

1.1 Problem

The "two-, three-, and four-level system" terminology for laser gain media suffices if the levels are well-separated compared to k_BT, or if some levels completely overlap. In low-quantum-defect systems, intermediate cases arise because of partial thermal population of the lower laser level or the upper pump level. Absorbing levels that are not well-separated compared to k_BT lead to ground state absorption at λ_L. Emitting levels that are not well-separated lead to absorption saturation at λ_P.

The "quasi-level" terminology is in wide use. Counting its occurrence in the title and abstract alone, 32 papers used "quasi-two, three, or four-level" in 2007. In the 10 years from 1997-2006, the terminology was used in the title or abstract of 197 papers (*1*). The terminology appears in the body of a far larger number of papers. Quasi-three-level is particularly ambiguous, because it could refer to systems that are either better or worse than "three-level." The aim of this technical report is to introduce a more quantitative terminology, useful for comparing various media, choices of pump and laser wavelengths, and different operating temperatures.

1.2 Results

Three quantities are identified that are figures of merit for the level structure of an ion, given a pump transition, and a laser transition. One quantity, ℓ, has a numerical value that corresponds closely to what one would expect for Nd-doped yttrium aluminum garnet (YAG) lasing at 1064 nm, the classic four-level system, and for $Cr:Al_2O_3$ lasing at 694.3 nm, the classic three-level system. A second quantity, f_0, is appropriate for characterizing the gain of an amplifying medium in the small signal regime. A third quantity, f_1, is appropriate for characterizing an amplifying medium in the large signal regime, e.g., a laser.

1.3 Conclusion

If the pumping involves only a single transition, and the lasing, as well, there is a straightforward definition of system level, ℓ, based on energy level structure and temperature, that spans the range 2-4. The occupancy factor f_0, and f_1, which range from –1 to +1, are figures of merit for the gain of a laser medium, under small signal and large signal conditions. All three quantities contain information that is complementary to the cross section. They are expected to be useful for comparing systems that (a) have a low quantum defect, i.e., they are pumped nearly in resonance with the laser wavelength, (b) are pumped with narrow band light, or in which (c) ground state absorption at the laser wavelength is a factor, or (d) stimulated emission (i.e., absorption saturation) at the pump wavelength is a factor.

2. Introduction

The terms "three-level-system" and "four-level-system" have been in wide use ever since the invention of the laser, and were used to describe masers even before that time. The introduction of a continuous "system level" scale would facilitate a quantitative comparison of laser media, operating temperatures, pump, and laser wavelengths. Several quantities are proposed below, based on the occupation probability of absorbing and emitting pump and laser levels. One quantity, ℓ, corresponds closely to what one would expect for Nd:YAG lasing at 1064 nm, the classic four-level system, and for Cr:Al2O3 lasing at 694.3 nm, the classic three-level system (2). A second quantity, f0, is appropriate for characterizing a gain medium used as an amplifier in the small signal regime. A third quantity, f_1, is appropriate for characterizing a gain medium used as an amplifier in the large signal regime, e.g., a laser.

These three quantities should be most useful in comparing systems that (a) have a low quantum defect, i.e., they are pumped nearly in resonance with the laser wavelength, (b) are pumped with narrow band light, or in which (c) ground state absorption at the laser wavelength is a factor, or (d) where stimulated emission at the pump wavelength is a factor. Such is the case for diode-pumped lasers based on Er^{3+}, Nd^{3+}, Yb^{3+}, and Ho^{3+}, which have been variously described as quasi-two-level, quasi-three-level, or quasi-four-level, depending on the particular transitions and temperature.

ℓ, f_0, and f_1 are indicators of suitability for light amplification and lasing, of importance equal to that of the cross section. Whether a particular transition will lase or not depends on other extrinsic factors, e.g., pump intensity, mirror reflectivities, etc. The point in this report is to isolate the statistical thermodynamic aspect of the gain medium, which depends explicitly on temperature and energy level alignments. In the next sections, we define the three quantities, and apply them to Er^{3+}, Nd^{3+}, and Yb^{3+} in YAG.

3. Methods, Assumptions, and Procedures

In a classic four-level laser (figure 1), in steady state, the relaxation rate, per electron, from the upper pump level to the upper laser level is fast compared to the thermal excitation rate, per electron, in the reverse direction, and typically fast compared to the rates of optical absorption and emission. Therefore the population in the upper laser level is large compared to the upper pump level, favoring the stimulated emission of laser photons over pump photons. The preferred energy level occupancy in the lower levels is the opposite, favoring the absorption of pump photons over laser photons. This situation holds over a variety of pump and laser transitions, and

over a broad range of temperature. Compared to three-level systems, four-level systems are easy to invert, because the upper laser level is easily populated, and the lower laser level is rapidly depopulated. This also means that the (unexcited) medium does not strongly absorb the laser wavelength, λ_L, when in the ground state.

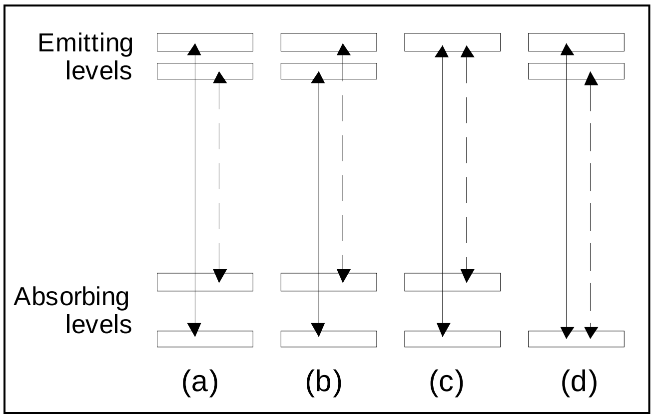

Figure 1. Different possibilities for four- and three-level systems, showing pump (solid) and laser (dashed) transitions. Intermediate cases, e.g. (b), can be quantitatively compared to the others, on the basis of the "system level", ℓ, and the occupation factors, f_0, and f_1.

In a classic three-level system (figure 1d), the lower laser level coincides with the ground state, e.g., in $Cr{:}Al_2O_3$ at 694.3 nm, in which case the unexcited medium absorbs at $\lambda_{L.}$ Alternatively, the upper pump level can coincide with the upper laser level (figure 1c), in which case the pump absorption saturates very easily. To distinguish between these two possibilities, one could refer to "two over one" systems, and vice versa. Other factors being equal, three-level systems are harder to invert than four-level systems.

In two-level systems, the initial state for emission of pump and laser photons is shared, as well as the initial states for absorption. These systems can only be inverted in steady state if the upper level has a higher degeneracy. We defer further consideration of degeneracy, because it will not change the substance of the conclusions below.

For purposes of discussion, we group together the absorbing states: all states that are close to the ground state (compared to the photon energies). Absorbing states within k_BT of the ground state can obviously be populated. Also grouped together are the highly excited states, i.e., those which can emit photons. In what follows, f_{eL} is the probability that the electron is in the initial state for emitting a Laser photon, given that it's in one of the emitting states. f_{aP} is the probability that the electron is in the initial state for absorbing a Pump photon, given that it's in one of the absorbing states.

4. Results and Discussion

4.1 Definition of "System Level", ℓ

We take as a premise that the level that absorbs (emits) pump photons contributes one to ℓ when it's full (empty); the level that emits (absorbs) laser photons contributes one to ℓ when it's full (empty). The proposed definition for the "system level" is

$$\ell = \frac{f_{eL}}{f_{eL} + f_{eP}} + \left(1 - \frac{f_{eP}}{f_{eL} + f_{eP}}\right) + \frac{f_{aP}}{f_{aP} + f_{aL}} + \left(1 - \frac{f_{aL}}{f_{aP} + f_{aL}}\right) \tag{1}$$

or, equivalently

$$\ell - 2 = \frac{2\left(f_{eL} f_{aP} - f_{eP} f_{aL}\right)}{\left(f_{eL} + f_{eP}\right)\left(f_{aP} + f_{aL}\right)}. \tag{2}$$

We will show below that the sign of $\ell - 2$ determines whether the photon flux of the laser beam increases at the expense of the pump beam, or vice versa.

In conventional usage, a three-level system does not become a two- or four-level system as the degree of excitation is varied. ℓ as defined by equation 1 satisfies this criterion, by construction. As it should, the definition depends on which of the two wavelengths is considered the laser. If the pump and laser wavelengths are exchanged, the new level number, ℓ', is given by $\ell + \ell' = 4$.

In the ideal four-level case, $f_{eL} = f_{aP} = 1$ and $f_{eP} = f_{aL} = 0$, therefore $\ell = 4$. For Nd:YAG pumped at 808 nm and lasing at 1064 nm, $\ell = 3.97$ at 300°K (see below). In the three-level case, the best scenario is $f_{eL} = f_{aP} = f_{aL} = 1$, $f_{eP} = 0$, therefore $\ell = 3$. For ruby pumped at 555 nm and lasing at 694.3 nm, $\ell = 3.00$ from 0 – 600°K. In the two-level case, the only scenario is $f_{eL} = f_{eP} = f_{aP} = f_{aL} = 1$, therefore $\ell = 2$. An example of an $\ell = 1$ system would be trying to lase ruby at 555 nm while pumping at 694.3 nm. An example of an $\ell = 0$ system would be trying to lase Nd:YAG at 808 nm by pumping at 1064 nm.

4.2 Definition of Occupation Factors f_0, and f_1

Consider the propagation of light at two wavelengths, coupled by a gain medium held at constant temperature. The rate equations for the total population density of absorbing states (N_1) and the total population of emitting states (N_2), at any point in the gain medium, include absorption and emission at both wavelengths [Lim2002, (*3*)].

$$\frac{dN_1}{dt} = +\Phi_P \sigma_P \left(f_{eP} N_2 - f_{aP} N_1 \right) + \Phi_L \sigma_L \left(f_{eL} N_2 - f_{aL} N_1 \right) + N_2 W_{21} \quad (3)$$

$$\frac{dN_2}{dt} = -\frac{dN_1}{dt} \quad (4)$$

Φ_P, and Φ_L are the pump and laser photon flux, σ_P and σ_L are the absolute cross section at the pump and laser wavelength, and W_{21} is the spontaneous emission rate. The total number of ions, $N_{tot} = N_1 + N_2$, is constant.

Neglecting the contribution of spontaneous emission, colinear laser and pump wavelengths will propagate according to

$$\frac{d\Phi_L}{dz} = \sigma_L \left(f_{eL} N_2 - f_{aL} N_1 \right) \Phi_L \quad (5)$$

$$\frac{d\Phi_P}{dz} = \sigma_P \left(f_{eP} N_2 - f_{aP} N_1 \right) \Phi_P \quad (6)$$

Solving equations 3 and 4 in steady state for N_1 and N_2, and substituting into equation 5, we obtain:

$$\frac{d\Phi_L}{dz} = \frac{\sigma_L \sigma_P N_{tot} \left(f_{eL} f_{aP} - f_{aL} f_{eP} - f_{aL} W_{21} / \Phi_P \sigma_P \right)}{\Phi_P \sigma_P \left(f_{eP} + f_{aP} \right) + \Phi_L \sigma_L \left(f_{aL} + f_{eL} \right) + W_{21}} \Phi_L \Phi_P . \quad (7)$$

When $\Phi_P \sigma_P , \Phi_L \sigma_L >> W_{21}$, we have

$$\frac{d\Phi_L}{dz} = \frac{\sigma_L \sigma_P N_{tot} \left(f_{eL} f_{aP} - f_{aL} f_{eP} \right)}{\Phi_P \sigma_P \left(f_{eP} + f_{aP} \right) + \Phi_L \sigma_L \left(f_{aL} + f_{eL} \right)} \Phi_L \Phi_P , \quad (8)$$

and

$$\frac{d\Phi_P}{dz} = -\frac{d\Phi_L}{dz} . \quad (9)$$

In view of equation 2, if $2 < \ell \le 4$, the photon flux in the laser beam increases at the expense of the pump beam, even if the pump is the weaker of the two. If $0 \le \ell < 2$, the opposite occurs, because equations 3 through 8 are symmetric with respect to interchange of pump and laser.

The normalization in equation 2 plays a role when there are more than two emitting levels, or more than two absorbing levels, i.e., in case $f_{eL} + f_{eP} < 1$ or $f_{aL} + f_{aP} < 1$. Note that the f dependence of $\ell - 2$ is not identical to that of the coupling in equations 8 and 9, nor is the quantity $\Lambda \equiv f_{eL} f_{aP} - f_{aL} f_{eP}$. One can have $\ell \sim 4$ even when the initial states for absorbing at

λ_P and emitting at λ_L are high in their respective manifolds, in which case the pump-laser coupling vanishes according to equations 8 and 9.

In special cases, the pump-laser coupling can be separated into a factor involving just cross sections and concentration, and a factor involving just occupancy. In an amplifier, for example, when $\Phi_L \sigma_L << \Phi_P \sigma_P$, Φ_L is effectively uncoupled from Φ_P, and grows exponentially according to

$$\frac{d\Phi_L}{dz} = \frac{\sigma_L N_{tot}\left(f_{eL} f_{aP} - f_{aL} f_{eP}\right)}{\left(f_{eP} + f_{aP}\right)} \Phi_L \,.$$

The part of the small signal gain coefficient that depends on occupancy is given by

$$f_0 = \frac{f_{eL} f_{aP} - f_{aL} f_{eP}}{f_{eP} + f_{aP}} \tag{10}$$

Inside a laser cavity, one may have $\Phi_L \sigma_L >> \Phi_P \sigma_P$, in which case the laser flux still grows if $\ell > 0$, but the gain is no longer exponential, and the laser is strongly coupled to the pump according to

$$\frac{d\Phi_L}{dz} = \frac{\sigma_P N_{tot}\left(f_{eL} f_{aP} - f_{aL} f_{eP}\right)}{\left(f_{aL} + f_{eL}\right)} \Phi_P \tag{11}$$

The part of the coupling coefficient that depends on occupancy is given by

$$f_1 = \frac{f_{eL} f_{aP} - f_{aL} f_{eP}}{f_{aL} + f_{eL}} \tag{12}$$

Thus different combinations of the occupancy factors are figures of merit in different situations. We focus on ℓ and f_0 in what follows.

4.3 Quasi-two-, Quasi-three-, and Quasi-four-Level Systems

Within a manifold, the occupation probability for a sublevel follows a Boltzmann distribution. For example,

$$f_{eL} = \exp\left(-E_{eL}\right) \Big/ \sum_e \exp\left(-E_e\right), \tag{13}$$

where the sum is over all the emitting states. In this case, the system level becomes

$$\ell = 2\left(\frac{1}{1+\exp\left(E_{eL} - E_{eP}\right)/kT} + \frac{1}{1+\exp\left(E_{aP} - E_{aL}\right)/kT}\right) \tag{14}$$

where E_{eL} is the energy of the initial state for emitting a laser photon, etc. The system levels thus calculated at 300°K are summarized in table 1. The levels which are not directly coupled by

photons serve as a reservoir for the active levels; but do not affect the system level. ℓ depends only on the difference between the pump and laser energy levels (*4*). In contrast, f_0 and f_1 depend on all of the energy levels, i.e., the complete level structure.

Table 1. System level ℓ, calculated for various gain media, wavelengths, and temperatures.

	λ_P	λ_L	**0°K**	**300°K**	**600°K**
Er:YAG	1470	1617	4.0	3.21	2.70
	1470	1645	4.0	3.29	2.79
	1532	1617	2.0	2.61	2.38
	1532	1645	2.0	2.70	2.47
Nd:YAG	808	1064	4.0	3.97	3.77
“	808	946	4.0	3.95	3.58
	869	946	4.0	3.17	2.87
	884	946	4.0	3.04	2.64
	886	946	3.0	2.94	2.70
Yb:YAG	941	1030	4.0	3.51	2.97

4.4 Er:YAG

In Er^{3+}, the *laser* transitions in the 1.6 µm region are between the $^4I_{15/2}$ (ground state) manifold and the $^4I_{13/2}$ (excited state) manifold (figure 2). For high-power, or low-quantum-defect applications, the *pump* transitions can also be between the $^4I_{15/2}$ manifold and the $^4I_{13/2}$. In the absence of upconversion, these are the only two manifolds that have significant occupation. Consideration of cross sections leads to pumping at 1470 nm or 1532 nm, and lasing at 1617 or 1645 nm (*5*,*6*).

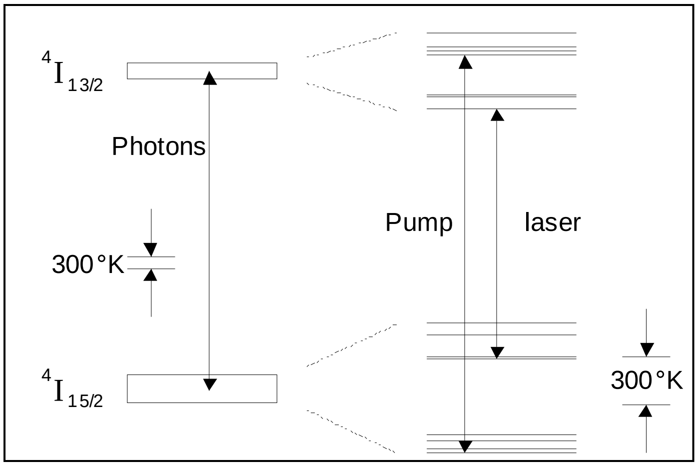

Figure 2. Er^{3+} ground state manifold and first excited state manifold. The scale on the right is magnified.

For λ_P = 1470 nm and λ_L = 1645 nm, the level alignment is favorable in both the absorbing states and the emitting states (figure 3a). That is, the lower laser level and the upper pump levels are high in their respective manifolds, and the upper laser level and lower pump levels are low in their respective manifolds. At 300°K, Er:YAG is effectively a 3.3-level system, according to equation 2. The contribution of the upper states is 1.9 and the contribution of the lower states is 1.4.

Neglecting the changes in the level energies, which are small in the 77 – 300°K range, we can easily plot the system level as a function of temperature (figure 3b). λ_L = 1617 behaves in a similar fashion. As the temperature decreases, the situation in both manifolds improves, reaching a system level of 4.0 at 70°K. f_0 is also plotted, and follows ℓ until the upper laser level population is frozen out. Of course, the absorption and emission cross sections will also change with temperature; their effect on lasing may even overwhelm that which is discussed here.

For λ_P = 1532 nm and λ_L = 1645 nm, the level alignment is favorable in the absorbing states, but not in the emitting states. At 300°K, ℓ = 2.7. Again neglecting the changes in the level energies, the situation in the upper manifold worsens as the temperature decreases. The optimum system level of 2.74 occurs at ~200°K (figure 4). This would be the optimum temperature for lasing only if all other factors were constant. f_0 closely follows ℓ throughout the temperature range.

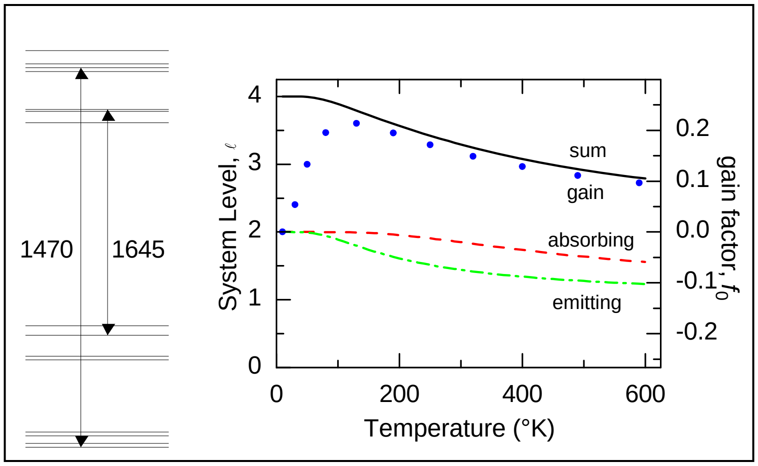

Figure 3. Er:YAG, λ_P = 1470 nm, λ_L = 1645 nm: (a) energy levels, (b) temperature dependence of the system level (sum), contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

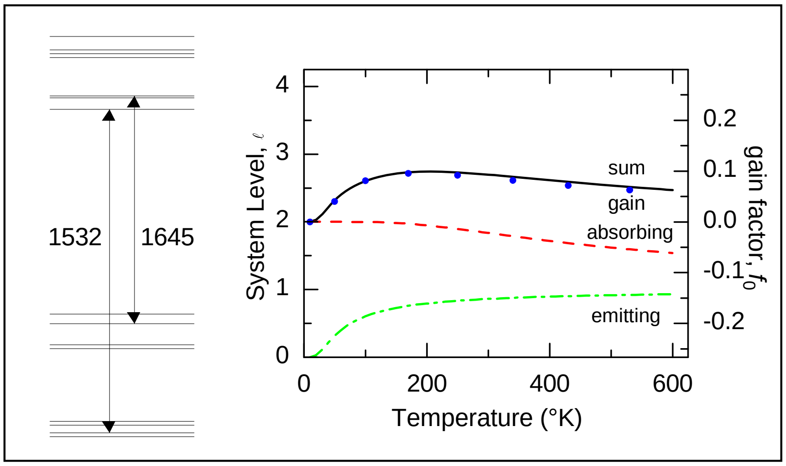

Figure 4. Er:YAG, λ_P = 1532 nm, λ_L = 1645 nm: (a) energy levels, (b) temperature dependence of the system level, contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

4.5 Nd:YAG

For λ_P = 808 nm, excitation occurs from the ground state $^4I_{9/2}$ to the $^4F_{5/2}$ manifold. The 1064 nm lasing transition involves two intermediate manifolds, the $^4F_{3/2}$ and the $^4I_{11/2}$. All four manifolds are well-separated, leading to $\ell \sim 4$ at all temperatures between 0-600°K. Other wavelengths can be made to lase by suppressing the four-level scheme (*7 through 17*). For λ_L = 941 nm, both absorbing levels are in the lowest manifold, but they are still well separated, so ℓ decreases only slightly.

To lower the quantum defect, it is possible to pump *and* lase between the $^4I_{9/2}$ and $^4F_{3/2}$ manifold (figure 5). When λ_P = 869 nm, and λ_L = 946 nm (*18*), the temperature has to be below ~30°K before the system has four-level character, because of the small splitting in the $^4F_{3/2}$ manifold. The system level decreases to $\ell \sim 3$ at 300°K. f_0 has a similar temperature dependence, because the initial states for pump absorption and laser emission are the lowest lying sublevels in their respective manifold.

When λ_P = 884.25 nm (*19*), the quantum defect is slightly lower, and ℓ decreases slightly as a result (figure 6). f_0, however, now decreases for temperatures below 240°K, because the occupancy of the lower pump level goes to zero.

When λ_P = 885.7 nm (*19*), the upper pump and laser levels are identical, so the system level never rises above $\ell \sim 3$, even at 0°K (figure 7). f_0 decreases at temperatures below 260°K, because the lower pump level is frozen out.

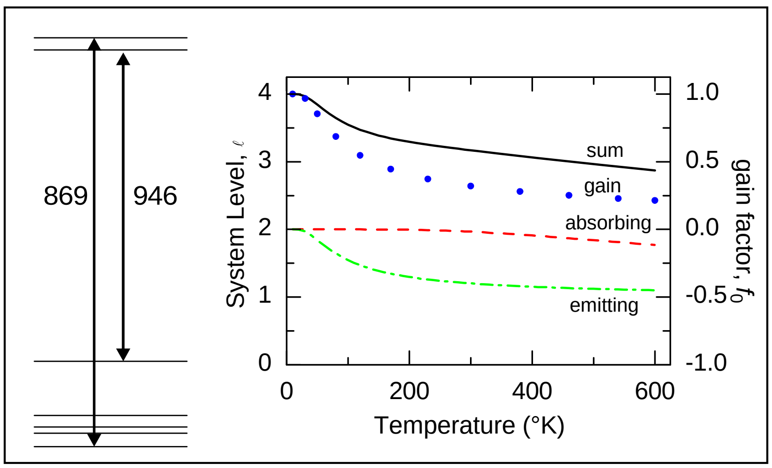

Figure 5. Nd:YAG, λ_P = 869 nm, λ_L = 946 nm: (a) energy levels, (b) temperature dependence of the system level, contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

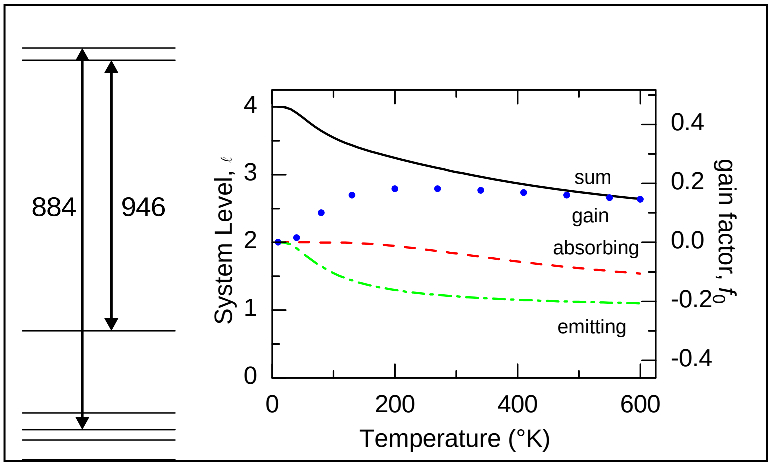

Figure 6. Nd:YAG, λ_P = 884 nm, λ_L = 946 nm: (a) energy levels, (b) temperature dependence of the system level, contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

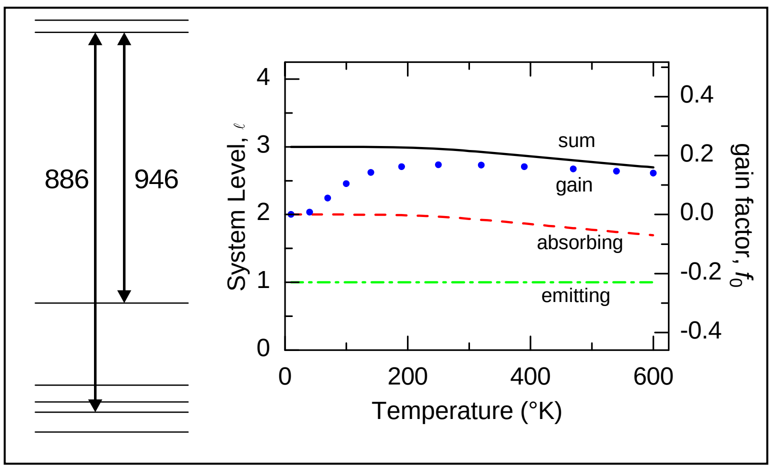

Figure 7. Nd:YAG, λ_P = 886 nm, λ_L = 946 nm: (a) energy levels, (b) temperature dependence of the system level, contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

4.6 Yb:YAG

In Yb:YAG, the $^2F_{7/2}$ and $^2F_{5/2}$ manifolds are the only energy levels involved in $4f - 4f$ transitions. The low quantum defect and the availability of diode laser pumping are well-known advantages in this system (*20 through 32*). However, the low quantum defect is reflected in a reduced ℓ at temperatures comparable to the quantum defect. When λ_P = 941 nm and λ_L = 1030 nm, the system level $\ell \sim 3.5$ at 300°K (figure 8). f_0 follows closely the temperature dependence of ℓ because f_{aP} and f_{eL} stay finite at low temperature, due to the initial states for pump absorption and laser emission being at the bottom of their respective manifolds.

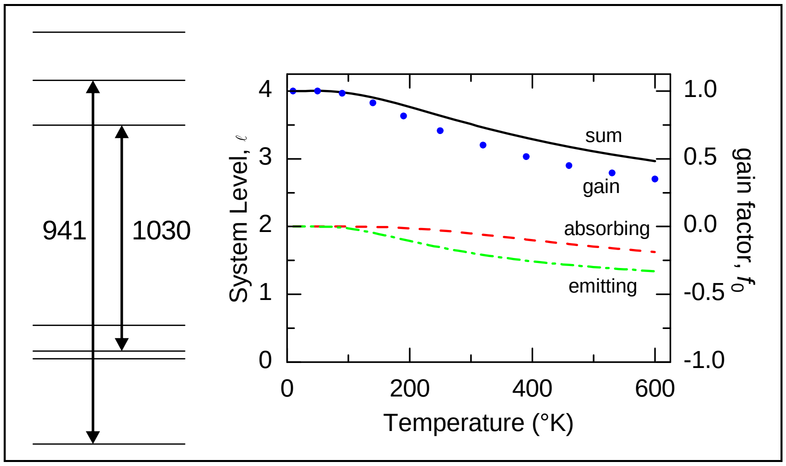

Figure 8. Yb:YAG, λ_P = 941 nm, λ_L = 1030 nm: (a) energy levels, (b) temperature dependence of the system level, contributions to system level by upper (emitting) and lower (absorbing) states, and the small signal gain factor f_0.

4.7 Discussion

The search for systems with a low quantum defect inevitably leads to a departure from the ideal case of a four-level-system. At some point, either ground state absorption at λ_L will become a factor, or absorption saturation at λ_P, or both. Often, the systems lie in between the classic two-level, three-level, and four-level cases. To quantitatively compare the different systems, and different temperatures, we have introduced a continuous system-level, ℓ, that corresponds closely to the classic three-level-laser, ruby at 694.3 nm, and the classic four-level-system, Nd:YAG at 1064 nm (if pumped in the visible or at 808 nm).

ℓ, f_0, and f_1 are all parameters complementary to the cross section, i.e., they are equally as important as the cross section, but they don't replace it. The three quantities have a similar, but distinct, dependence on the occupancy factors. ℓ is best at quantifying the commonly used terminology for "three-level-system," etc. f_0 is most appropriate for describing the small signal gain of an amplifier. f_1 is most appropriate for describing the coupling between pump and signal within a laser cavity.

The calculations of ℓ presented here assume that the energy level structure between 0 – 600°K is close to that measured at 300°K. Continuous energy level data for Er^{3+}, Nd^{3}+, Yb^{3+} from 0 – 600°K are not available. The energy levels are typically only measured at a few discrete temperatures, e.g., 4.2°K, 77°K, and 300°K. In these cases, the energies do not shift significantly.

It is interesting to note that, for all the cases considered here, except ruby, the absorbing (lower) states contribute more to the system level than the emitting (upper) states.

Quasi-three-level systems have been previously analyzed for situations that are much more complicated than the simple one in section 4.2. The closest treatments are described below. An analysis combining rate equations and plane-wave propagation for a pump pulse has been presented for quasi-three-level energy storage lasers and amplifiers, e.g., Yb:YAG pumped at 941 nm, and at 968 nm (*20*). Another analysis of quasi-three-level lasers considered the broadband regime, where the pump induced transitions between all states of two manifolds, using Tm:YAG as an example (*33*). Quasi-three-level end-pumped lasers have been analyzed in the time-dependent Q-switched regime, where the pump and laser are not simultaneously present (*34*). A cw model of quasi-three-level lasers includes the same four occupancy factors, but does not identify a quantity like ℓ (*35*). A cw model of quasi-three-level lasers did not include saturation at λ_P, but extended the usual analysis to an arbitrary distribution of pump and laser spatial modes (*24*). A theoretical and experimental investigation of a diode-pumped quasi-three-level laser used a rate equation analysis and considered the propagation of Gaussian pump and laser beams in Yb:BdCOB, taking into account the effects of absorption saturation, temperature profile, and beam quality factor of the pump diode (*36*). The factor Λ has been included in prior analyses, but not expounded upon. For example, an analysis of end-pumped quasi-three-level lasers, focused on laser output, calculating the optimum output mirror reflectivity, crystal length, doping and temperature, using Yb:YAG as an example (*27*).

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5. Conclusion

The "two-, three-, and four-level system" terminology is widely used to describe lasers. The terminology suffices if the levels are well-separated compared to k_BT, or if some levels completely overlap. In low-quantum-defect systems, intermediate cases arise because of partial thermal excitation of the lower laser level or the upper pump level. If the pumping involves only a single transition, and the lasing as well, there is a straightforward definition of system level, based on energy level structure and temperature, that spans the range 2-4. The occupancy factors f_0, and f_1, which range from –1 to +1, are figures of merit for the gain of a laser medium, under small signal and large signal conditions. All three quantities are complementary to the cross section. They are expected to be useful for comparing systems that (a) have a low quantum defect, i.e., they are pumped nearly in resonance with the laser wavelength, (b) are pumped with narrow band light, or in which, (c) ground state absorption at the laser wavelength is a factor, or (d) stimulated emission at the pump wavelength is a factor.

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